To illustrate the above the dimensionless temperature θ is shown against the dimensionless time Fo with parameters, namely, the value of the relaxation function H_0 of the internal energy at a current instant, and the dimensionless relaxation time Fo_1. In the classical heat-conduction theory one has $H_0 = 1$ and $Fo_1 = 0$. Therefore, as seen from Fig. 1, when H_0 is suitably high, say, $H_0 = 0.95$, a change in Fo_1 has a slight effect on the results. Similarly, for $Fo_1 \to 0$ a variation in the parameter H_0 has no noticeable effect on the nonstationary temperatures. However, for small values of H_0 and large Fo_1 there is a considerable effect of the fading memory. This is clear from the diagram for $H_0 = 0.1$ and $Fo_1 = 100$.

NOTATION

T, temperature; T_0, equilibrium temperature; T_f, temperature of the surrounding medium; M, point of volume; N, point of surface; s, integration variable; p, Laplace variable; h(s), relaxation function of internal energy; $\lambda(s)$, relaxation function of heat flow; α, heat-exchange coefficient; τ, time; Fo, dimensionless time; Fo_s, dimensionless integration variable; H_0, the value of dimensionless relaxation function of internal energy at current time; H(s), dimensionless relaxation function of internal energy; θ, dimensionless temperature; Fo_1, dimensionless relaxation time of internal energy; θ_f, dimensionless temperature of the medium.

LITERATURE CITED

1. A. V. Lykov, Inzh.-Fiz. Zh., 26, No. 5, 781 (1974).
2. I. W. Nunziato, Quart. Appl. Math., 187 (1971).
3. G. E. Shilov, Lectures on Vector Analysis [in Russian], Gostekhizdat, Moscow (1954).
4. G. A. Korn and T. M. Korn, Mathematical Handbook for Scientists and Engineers, 2nd ed., McGraw-Hill (1967).

NONSTATIONARY FILTRATION OF A THREE-PHASE MIXTURE TAKING ACCOUNT OF GRAVITATION

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A method of solution and results of calculations are presented for the problem of displacement of gasified petroleum by water in an inclined stratum.

The process of displacement of gasified petroleum by water in an inclined stratum which is assumed homogeneous is examined in this paper; the physical properties of the fluids and collector are considered known. It is also kept in mind that the process is isothermal and that thermodynamic equilibrium is built up instantaneously between coexisting phases. We neglect the influence of capillary forces.

Under the assumptions mentioned, the process of one-dimensional nonstationary filtration of a three-phase mixture (water — petroleum — gas) is described by a nonlinear system of second-order partial differential equations (see [1]), which is written as follows in dimensionless form:

$$\frac{\partial}{\partial x}\left[\frac{k_p s_p}{\mu_p \beta_p}\left(\frac{\partial p}{\partial x}+a_p\right)+\frac{k_g \mu_2}{\mu_g \beta_g}\left(\frac{\partial p}{\partial x}+a_g \gamma_g\right)\right]=\frac{\partial}{\partial t}\left[\frac{(1-\sigma_g-\sigma_w)\, s_p}{\beta_p}+\frac{\sigma_g}{\beta_g}\right],$$

$$\frac{\partial}{\partial x}\left[\frac{k_p}{\mu_p \beta_p}\left(\frac{\partial p}{\partial x}+a_p\right)\right]=\frac{\partial}{\partial t}\left(\frac{1-\sigma_g-\sigma_w}{\beta_p}\right), \qquad (1)$$

$$\frac{\partial}{\partial x}\left[\frac{k_w \mu_1}{\mu_w \beta_w}\left(\frac{\partial p}{\partial x}+a_w\right)\right]=\frac{\partial}{\partial t}\left(\frac{\sigma_w}{\beta_w}\right).$$

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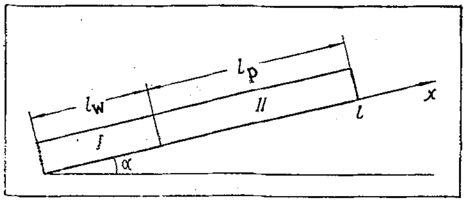

Fig. 1. Diagram of working the stratum.

The following are selected as the characteristic values to which the dimensional quantities are referred: p_s, k, μ_{ps}, μ_{ws}, μ_{gs}, γ_{gs}, L, and the characteristic time $t* = m\mu_{ps}L^2/(kp_s)$. The quantities a_p, a_g, a_w have the form $a_p = \gamma_p L \sin\alpha/p_s$, $a_g = \gamma_{gs} L \sin\alpha/p_s$, $a_w = \gamma_w L \sin\alpha/p_s$. These are dimensionless parameters which characterize the influence of gravitation. Here $p = p(x, t)$, $\sigma_g = \sigma_g(x, t)$, $\sigma_w = \sigma_w(x, t)$ are the desired functions, while k_p, k_g, k_w are given functions of the saturation of σ_g and σ_w, and s_p, β_p, β_g, μ_p, γ_g are given functions of the pressure p.

The numerical solution of filtration problems described by a system of equations of the type (1) has been considered in [2, 3], for example. A numerical method based on the transformation of (1) proposed in [4] and yielding a saving in machine time is proposed for the solution of the problem posed in this paper. Consequently, changes in the characteristics of the process in both the time and in the length of the stratum have been obtained.

Let us consider the process of working a thin inclined linear stratum of length l in the mixed mode of displacing gasified petroleum by water under contour flooding conditions while maintaining a constrant drop between the pressure and utilization galleries (Fig. 1).

Prior to the beginning of utilization the pressure in the stratum was distributed only under the influence of gravitational forces. The mixture consisted of two phases: water (domain I in Fig. 1) and petroleum (domain II in Fig. 1). The saturation of the phases in domains I and II is constant, while there is a jump in saturation on their interface. At the beginning of utilization the pressure in the utilization gallery is instantly reduced to the given pressure and is then kept constant. The pressure at which pumping of the water occurs is also constant. The pressure in the stratum will drop upon sampling the mixture. This results in degasification of the petroleum in domain II and the appearance of a gas phase. A mixed mode of three-phase mixture filtration arises. This process is described by the system (1) under the following initial and boundary conditions

$$p(x, 0) = \begin{cases} 1 + a_p l_p + a_w (l_w - x), & 0 \leqslant x \leqslant l_w, \\ 1 + a_p (l - x), & l_w \leqslant x \leqslant l, \end{cases}$$
$$\sigma_g(x, 0) = 0, \quad 0 \leqslant x \leqslant l, \tag{2}$$
$$\sigma_w(x, 0) = \begin{cases} 0.85, & 0 \leqslant x \leqslant l_w, \\ 0.2, & l_w \leqslant x \leqslant l, \end{cases}$$

$$p(0, t) = 1 + a_p l_p + a_w l_w, \quad \sigma_g(0, t) = 0,$$
$$\sigma_w(0, t) = 0.85, \quad p(l, t) = p_c \text{ (const)}. \tag{3}$$

Let us introduce notation for the desired functions: $u = \sigma_g$, $v = \sigma_w$. Let us write the system (1) in the matrix form

$$\frac{\partial C(y)}{\partial t} = \frac{\partial}{\partial x}\left(A(y)\frac{\partial p}{\partial x} + B(y)\right), \tag{4}$$

where

$$y = (p, u, v), \quad C = (c_1, c_2, c_3), \quad A = (a_1, a_2, a_3), \quad B = (b_1, b_2, b_3),$$
$$c_1 = \frac{(1 - u - v)s_p}{\beta_p} + \frac{u}{\beta_g}, \quad c_2 = \frac{1 - u - v}{\beta_p}, \quad c_3 = \frac{v}{\beta_w},$$
$$a_1 = \frac{k_p s_p}{\mu_p \beta_p} + \frac{k_g \mu_2}{\mu_g \beta_g}, \quad a_2 = \frac{k_p}{\mu_p \beta_p}, \quad a_3 = \frac{k_w \mu_1}{\mu_w b_w},$$
$$b_1 = \frac{a_p k_p s_p}{\mu_p \beta_p} + \frac{a_g \gamma_g k_g \mu_2}{\mu_g \beta_g}, \quad b_2 = \frac{a_p k_p}{\mu_p \beta_p}, \quad b_3 = \frac{a_w k_w \mu_1}{\mu_w \beta_w}.$$

Let us perform the transformation of the system (4) to an equivalent form as proposed in [4].

We assume that the Jacobian $J = \partial(c_1, c_2, c_3)/\partial(p, u, v)$ does not vanish in the domain under consideration. Then $Q = \|q_{ij}\| = (\partial C/\partial y)^{-1}$ exists. We represent the left side of the system (4) in the form

$$\frac{\partial C(y)}{\partial t} = \frac{\partial C}{\partial y} \frac{\partial y}{\partial t}.$$

Multiplying both sides of this system by the matrix Q, we obtain

$$\frac{\partial y}{\partial t} = Q \frac{\partial}{\partial x} \left(A \frac{\partial p}{\partial x} + B\right).$$

We consider the first equation of this system as an equation for p, namely,

$$\frac{\partial p}{\partial t} = \sum_{j=1}^{3} q_{1j} \frac{\partial}{\partial x} \left(a_j \frac{\partial p}{\partial x} + b_j\right). \tag{5}$$

We obtain the equations for u and v as follows. We rewrite the system (4) as

$$\frac{\partial C}{\partial t} = A \frac{\partial^2 p}{\partial x^2} + \frac{\partial A}{\partial x} \frac{\partial p}{\partial x} + \frac{\partial B}{\partial x}. \tag{6}$$

By eliminating $\partial^2 p/\partial x^2$ two independent equations which comprise the desired system for u and v must be obtained from the three equations in the system (6). Let us note that the functions k_p, k_g, k_w are such that they, and therefore also the coefficients a_j, b_j, vanish at some portions of the segment $0 \leq x \leq l$. Hence, it is impossible to eliminate $\partial^2 p/\partial x^2$ from the two remaining equations by using any one equation of the system (6) in the whole segment $0 \leq x \leq l$, since a degenerate system of equations will be obtained because of such an elimination. We proceed as follows. Noting that $a_1 + a_2 + a_3 > 0$ everywhere for $0 \leq x \leq l$, we consider a linear combination of Eqs. (6) which will be the sum of these equations. We eliminate $\partial^2 p/\partial x^2$ from all the equations of the system (6) by using this linear combination. It can be shown that any two equations hence obtained will be linearly independent and therefore can be considered as a system of equations in u and v. Hence, by transforming the first two equations of (6), for example, in such a manner, we obtain the following system for u and v:

$$\begin{aligned} a_1 \frac{\partial (c_2 + c_3)}{\partial t} - (a_2 + a_3) \frac{\partial c_1}{\partial t} &= \left[a_1 \frac{\partial (a_2 + a_3)}{\partial x} - (a_2 + a_3) \frac{\partial a_1}{\partial x} \right] \frac{\partial p}{\partial x} + a_1 \frac{\partial (b_2 + b_3)}{\partial x} - (a_2 + a_3) \frac{\partial b_1}{\partial x}, \\ a_2 \frac{\partial (c_1 + c_3)}{\partial t} - (a_1 + a_3) \frac{\partial c_2}{\partial t} &= \left[a_2 \frac{\partial (a_1 + a_3)}{\partial x} - (a_1 + a_3) \frac{\partial a_2}{\partial x} \right] \frac{\partial p}{\partial x} + a_2 \frac{\partial (b_1 + b_3)}{\partial x} - (a_1 + a_3) \frac{\partial b_2}{\partial x}. \end{aligned} \tag{7}$$

Thus, we have gone from the system (4) to its equivalent system (5) and (7). Let us consider a difference approximation of the problem (5), (7), (2), (3). Let h and τ be the spacings in x and t:

$$p_i^n = p(x_i, t_n), \; u_i^n = u(x_i, t_n), \; v_i^n = v(x_i, t_n),$$
$$x_i = ih, \; i = 0, 1, \ldots, N, \; x_N = l, \; t_n = n\tau, \; n = 0, 1, \ldots,$$
$$a_{j,i}^n = a_j(p_i^n, u_i^n, v_i^n), \; b_{ji}^n = b_j(p_i^n, u_i^n, v_i^n),$$
$$c_{j,i}^n = c_j(p_i^n, u_i^n, v_i^n), \; q_{1j,i}^n = q_{1j}(p_i^n, u_i^n, v_i^n),$$
$$j = 1, 2, 3.$$

Let us approximate (5) by the following scheme:

$$\frac{p_i^{n+1} - p_i^n}{\tau} = \frac{1}{h} \sum_{j=1}^{3} q_{1j,i}^n \left[a_{j,i+1/2}^n \frac{p_{i+1}^{n+1} - p_i^{n+1}}{h} - a_{j,i-1/2}^n \frac{p_i^{n+1} - p_{i-1}^{n+1}}{h} + \frac{b_{j,i+1}^n - b_{j,i-1}^n}{2} \right], \; i = 1, 2, \ldots, N-1. \tag{8}$$

Here $a_{i\pm1/2} = {}^1/_2(a_i + a_{i\pm1})$.

From the boundary conditions we have

$$p_0^{n+1} = 1 + a_p l_p + a_w l_w, \; p_N^{n+1} = p_c. \tag{9}$$

The order of the approximation of the scheme (8) and (9) is $O(\tau + h^2)$.

A factorization method is used to solve the system (8) and (9). As is known (see [5], for example), compliance with the condition

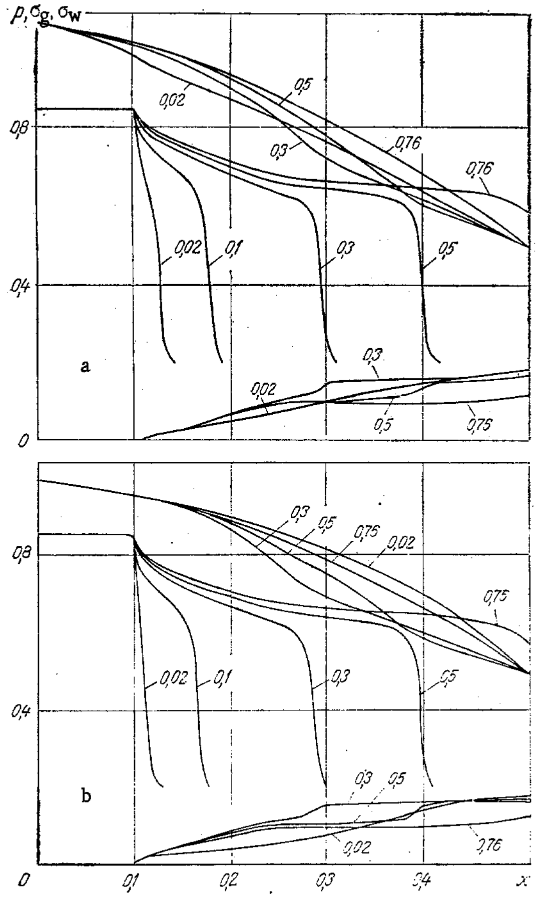

Fig. 2. Distribution of p (upper series of curves), σ_g (lower), and σ_w (middle) along the length of an inclined stratum (a) and for a horizontal stratum (b); numbers on the curves signify the values of t.

$$\sum_{j=1}^{3} q_{1j,i}^{n} a_{j,i}^{n} > 0$$

everywhere for $0 \le x \le l$ is sufficient for stability of this method. As can be verified, this condition holds for the problem under consideration.

Now let us turn to the system (7) by considering it relative to u and v. It can be seen by direct substitution that it is hyperbolic and that both families of characteristic directions of this system have a slope to the right so that the derivatives with respect to x should be replaced by left differences (see [6]) in order to obtain a stable scheme. According to the above, we consider the following implicit difference scheme for (7) (for brevity, we omit the superscript n + 1, for example, $a_{j,i}$ denotes $a_{j,i}^{n+1}$):

$$\begin{gathered}
a_{1,i}\,\frac{c_{2,i}+c_{3,i}-c_{2,i}^{n}-c_{3,i}^{n}}{\tau}-(a_{2,i}+a_{3,i})\,\frac{c_{1,i}-c_{1,i}^{n}}{\tau}= \\
=\left[a_{1,i}\,\frac{a_{2,i}+a_{3,i}-a_{2,i-1}-a_{3,i-1}}{h}-(a_{2,i}+a_{3,i})\,\frac{a_{1,i}-a_{1,i-1}}{h}\right]\times \\
\times\frac{p_i-p_{i-1}}{h}+a_{1,i}\,\frac{b_{2,i}+b_{3,i}-b_{2,i-1}-b_{3,i-1}}{h}-(a_{2,i}+a_{3,i})\,\frac{b_{1,i}-b_{1,i-1}}{h}, \\
a_{2,i}\,\frac{c_{1,i}+c_{3,i}-c_{1,i}^{n}-c_{3,i}^{n}}{\tau}-(a_{1,i}+a_{3,i})\,\frac{c_{2,i}-c_{2,i}^{n}}{\tau}= \\
=\left[a_{2,i}\,\frac{a_{1,i}+a_{3,i}-a_{1,i-1}-a_{3,i-1}}{h}-(a_{1,i}+a_{3,i})\,\frac{a_{2,i}-a_{2,i-1}}{h}\right]\times
\end{gathered} \tag{10}$$

$$\times \frac{p_i - p_{i-1}}{h} + a_{2,i} \frac{b_{1,i} + b_{3,i} - b_{1,i-1} - b_{3,i-1}}{h} -$$
$$-(a_{1,i} + a_{3,i}) \frac{b_{2,i} - b_{2,i-1}}{h}, \quad i = 1, 2, \ldots, N.$$

From the boundary conditions we obtain

$$u_0^{n+1} = 0, \quad v_0^{n+1} = 0.85. \tag{11}$$

The value of p_i calculated in the problem (8) and (9) for the (n + 1)-th layer is used as p_i in (10).

The nonlinear system of equations (10) and (11) was solved as follows. For each fixed i, starting with i = 1 to i = N, Eqs. (10) were considered as a system of nonlinear equations in u_i^{n+1}, v_i^{n+1}, whose solution was found by iterations by the Newton method. The values $u_{i-1}^{n+1} - u_{i-1}^n + u_i^n$, $v_{i-1}^{n+1} - v_{i-1}^n + v_i^n$ were taken as good initial approximations.

The difference method considered yielded good results. On the average, 2-3 iterations by the Newton method assured sufficient accuracy.

The difficulty arising in this problem should be noted, which is that the functions s_p, β_p, μ_p are such that they become constants at a pressure greater than the saturation pressure. This means that the Jacobian J vanishes for p > 1, which complicates the application of the method described. Hence, a matrix adjoint to the matrix $\partial C/\partial y$ (i.e., comprised of complements to the elements of the latter) was taken as Q in the actual computation, and the equation

$$J \frac{\partial p}{\partial t} = \sum_{j=1}^{3} q_{1j} \frac{\partial}{\partial x}\left(a_j \frac{\partial p}{\partial x} + b_j \right)$$

was considered in place of (5).

Let us present an example of the computation. The computation was performed for a stratum with the following parameters: L = 500 m, $l_p = 0.4$, $l_w = 0.1$, m = 0.2, k = 0.2d, $p_s = 140$ kg/cm², $p_c = 0.5$. The relative phase permeabilities were determined by the relationships (see [7])

$$k_p = \begin{cases} \left(\frac{0.85 - \sigma_g - \sigma_w}{0.85} \right)^{2.8} (1 + 2.4\sigma_w + 16.5\sigma_g\sigma_w), & \sigma_g + \sigma_w \leqslant 0.85, \\ 0, & 0.85 \leqslant \sigma_g + \sigma_w \leqslant 1, \end{cases}$$

$$k_g = \begin{cases} \left(\frac{\sigma_g - 0.1}{0.9} \right)^{3.5} (4 - 3\sigma_g), & 0.1 \leqslant \sigma_g \leqslant 1, \\ 0, & 0 \leqslant \sigma_g \leqslant 0.1, \end{cases}$$

$$k_g = \begin{cases} \left(\frac{\sigma_w - 0.2}{0.8} \right)^{3.5}, & 0.2 \leqslant \sigma_w \leqslant 1, \\ 0, & 0 \leqslant \sigma_w \leqslant 0.2. \end{cases}$$

The viscosities of the phases and their specific gravities were considered equal $\mu_{ps} = 1.6$ cp, $\mu_{gs} = 0.01$ cp, $\mu_{ws} = 1$ cp, $\mu_g = 1$, $\mu_w = 1$, $\gamma_p = 0.85 \cdot 10^{-3}$ kg/cm³, $\gamma_w = 1.2 \cdot 10^{-3}$ kg/cm³, $\gamma_{gs} = 0.28 \cdot 10^{-3}$ kg/cm³.

The pressure dependences of the physical properties of the petroleum and gas were expressed by the following formulas (see [1]):

$$s_p(p) = \begin{cases} 66p + 10.297, & 0.214 \leqslant p \leqslant 1, \\ 76.297, & p \geqslant 1, \end{cases}$$

$$\mu_p(p) = \begin{cases} 1.5 - 0.5p, & 0.214 \leqslant p \leqslant 1, \\ 1, & p \geqslant 1, \end{cases}$$

$$\beta_p(p) = \begin{cases} 0.177p + 1.027, & 0.214 \leqslant p \leqslant 1, \\ 1.204, & p \geqslant 1, \end{cases}$$

$$\beta_g(p) = 0.007/p, \quad \gamma_g(p) = p, \quad \beta_w = 1.$$

For these values of the parameters 132 days correspond to the unit of dimensionless time.

The results of computations are presented in Fig. 2a and b. Shown is the pressure and saturation distributions along the stratum at different times t, where Fig. 2a corresponds to an inclined stratum ($\alpha = 30°$), and Fig. 2b to a horizontal stratum ($\alpha = 0°$).

The results obtained correspond to existing conceptions. The form of the pressure distribution curves for different times is due to the processes of petroleum degasification, and petroleum and gas displacement by water, and also by the compressibility of the gas. The shape of the σ_g distribution curves reflects the initial growth of σ_g because of degasification of the petroleum and broadening of the moving gas domain, and then the diminution in σ_g to the critical value at which it is immobile. This occurs because the mobile gas is displaced by the water being introduced into the stratum. The difference between the σ_g curves in Fig. 2a and b for the same values of t is explained by the fact that the gas being separated out of the petroleum under the effect of gravitational forces in an inclined stratum will migrate upward into the utilization gallery. Hence, for example, for t = 0.3, 0.5 and 0.76 the mobile gas zone in the stratum and the value of σ_g in this zone are somewhat less for an inclined stratum than for a horizontal stratum.

NOTATION

k_p, k_g, k_w, relative phase permeabilities for petroleum, gas, and water, respectively; p, pressure; p_s, saturation pressure; σ_g, σ_w, gas and water saturation; μ_p, μ_g, μ_w, phase viscosities; μ_{ps}, μ_{gs}, μ_{ws}, phase viscosities at $p = p_s$; $\mu_1 = \mu_{ps}/\mu_{ws}$, $\mu_2 = \mu_{ps}/\mu_{gs}$; β_p, β_g, β_w, volume phase coefficients; s_p, gas solubility in petroleum; γ_p, γ_g, γ_w, phase specific gravities; γ_{gs}, gas specific gravity at $p = p_s$; k, absolute permeability of the medium; m, porosity; t, time; x, space coordinate; L, length of the stratum; α, angle of inclination of the stratum to the horizon.

LITERATURE CITED

1. M. D. Rozenberg, S. A. Kundin, A. K. Kurbanov, N. I. Suvorov, and G. Yu. Shovkrinskii, Filtration of a Gasified Fluid and Other Multicomponent Mixtures in Petroleum Strata [in Russian], Nedra, Moscow (1969).
2. V. F. Baklanovskaya, S. A. Kundin, A. K. Kurbanov, M. D. Rozenberg, I. I. Chechel', and G. Yu. Shovkrinskii, in: Scientific Technical Conference on Oil Recovery [in Russian], No. 40, Nedra, Moscow (1971), p. 18.
3. V. F. Baklanovskaya, S. A. Kundin, A. K. Kurbanov, M. D. Rozenberg, and I. I. Chechel', in: Proceedings of the All-Union Oil—Gas Scientific Research Institute "Development of Petroleum Deposits and Hydrodynamics of a Stratum" [in Russian], No. 60, Nedra, Moscow (1974), p. 17.
4. L. F. Yukhno, Zh. Vychisl. Mat. Mat. Fiz., No. 4, 977 (1975).
5. A. A. Samarskii, Introduction to the Theory of Difference Schemes [in Russian], Nauka, Moscow (1971).
6. B. L. Rozhdestvenskii and N. N. Yanenko, Systems of Quasilinear Equations [in Russian], Nauka, Moscow (1968).
7. I. A. Charnyi, Underground Hydrogasdynamics [in Russian], Gostoptekhizdat, Moscow (1963).